XPath: (P)DL on trees.

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Overview

1. KR on the Web

2. Logical research questions for XML

3. Getting familiar with XPath(s)

4. Zoom in
   i. Expressivity
   ii. Complexity

5. Conclusions
KR on the Web

**ABS2000** Edge labelled graphs queried by regular path expressions

**XML** Node labelled *sibling ordered* trees queried by XPath

**RDF** triples and non wellfounded sets

• ... but most web information is of course in the form of ...
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RDF triples and non wellfounded sets

- ... but most web information is of course in the form of ... text
  sometimes generated from a relational database.

- This talk: XML.
Graphs and trees

- Edge labelled graphs can very directly encode ER diagrams.
- These can always be represented as trees.
- Choice of representation determines
  - query processing costs
  - needed expressive power for your
    - query language
    - constraint language
  - robustness for changes in the data-structures
Example: interviews

- Sigmod Record Distinguished DB Profiles

- Simple model:

  An interview consists of a list of questions each followed by a list of answers.
XPath: (P)DL on trees.

exemelize this
In practice

pdftohtml -xml |
saxon MakeInterviewTree.xsl >> interview.xml

Quiztime

1. How will the output of pdftohtml look as a tree?

2. What will be the easiest (and fastest) tree transformation?

3. Which of the 4 tree models?
In theory: TREE model

- **Query:** give me all QA pairs.
In theory: TREE model

- Query: give me all QA pairs.

- In “hybrid DL”:

  - for $q$ such that $q \models Q$, return
    \[
    (q, \{ a \mid a \models A \land \exists.\text{parent } q \})
    \]
In theory: TREE model

- In XPath 2.0:

  - for $q$ in //Q return ($q,$q/A)
In theory: TREE model

- In XPath 2.0:
  - for $q$ in //Q return ($q,$q/A)

In DL you describe the node that you want, in XPath you tell how to get there.
Same query on the practical FLAT model

• **Query**: return all A-nodes answering a give Q node

• **Tree model**: simple ALC-formula using the tree-order

• **Flat tree model**:
  - use the document-order or the sibling-order
  - all A nodes *after* the given Q, but *before* the next Q
  - 3 variables …
  - not modally expressible …
  - the wanted A-nodes must satisfy \( A \land \text{since}(q, \neg Q) \)
Constraining the models: theory vs practice

- XML constraint languages are based on tree-automata
- languages use regular expressions over node-labels.
- these describe the children of a node read from left to right

Flat model  interview --> (Q,A+)+

Tree model  interview --> Q+.  Q --> A+

Data  Actual question and answer text is stored in attribute nodes.
Constraining the models: theory vs practice: robustness

- **Example:** Extend our constraints: every interview ends with a bye-bye question which receives no answer.

- In all models this is expressible as a FO sentence: thus a regular tree language.

**New Flat model** Easy: interview → (Q,A+)+,Q

**New Tree model** Hard! Not expressible by a DTD. (Proof later)
Bad!

- Difficult to accept and understand non-expressibility by practitioners
- leads to underspecified documents
- leads to frustration and unsafe coupling
New Tree model

- We need types to express the last answerless question.

- Specialized DTD’s = MSO = regular tree languages  
  [Papakonstantinou, Vianu 00]

- NormalQ and EndQ are types of Q

- interview \rightarrow NormalQ+, EndQ

- NormalQ \rightarrow A+

- EndQ \rightarrow EMPTY
New Tree model

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- Specialized DTD’s = MSO = regular tree languages
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- This is not expressible in XML Schema!
KR on the web: wrap up

- Most information on the web is in implicitly structured text.
- Asking complex queries to the web thus means to extract and make this structure explicit.
- This often leads to rather flat (“reading text-ordered”) XML.
- KR languages are important to describe, constrain and validate the XML,
- because these XML files are themselves often input to other knowledge-extraction programs (tree-transformations, queries)
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**XML-tasks**

[Schwentick 04] distinguishes the following four:

- Validation
- Transformation
- Navigation
- Querying

Every task must be described in some (logical) language.
Usual research questions

Given some language $L$

- What tasks can I express in $L$? How well can I express them in $L$?
- Given an $L$ expression and data, what are the computation costs to perform the task?
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Given some language $L$

- What tasks can I express in $L$? How well can I express them in $L$?

- Given an $L$ expression and data, what are the computation costs to perform the task?

- Each task may involve more specialized questions: e.g.

- **Typechecking**: given input conform $I_1 \in L_1$, given a transformation $T \in L_T$, will the output always be conform $I_2 \in L_2$?

- [Milo, Suciu, Vianu, 00] Decidable for DTD and Core XSLT.
This talk: focus on validation and navigation

Expressive power on trees

- relative to yardsticks as CQ, FO, MSO, tree automata
- semantic characterizations
- succinctness questions
- rewrite systems
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- semantic characterizations
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Complexity

- **Model checking**: given a tree $T$ and a formula $F$, does $T$ satisfy $F$?
- **Static analysis**: containment, equivalence, satisfiability of expressions.
Major techniques and strategies

- Similar research strategy as in DL: understand a language landscape by asking the same question for many different fragments.

- Where are the borders of decidability and tractability?

- Develop handy tools to show that something is not expressible in some fragment.

- Techniques include
  - Finite models
  - Tree automata, regular tree languages
  - tree decompositions
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XPath

- **two sorted language**, just as (P)DL
  - path sort binary relation between nodes
  - node sort set of nodes

- interpreted on a special class of models:
  - finite, sibling ordered, node-labelled unranked trees

- XPath, like DL, is not a language, more a “style”, a “family”
Operators on node sort are very familiar

- atomic tests
- test for being in the domain of a relation. (just like $\exists R. F$)
- closed under the booleans.
- (sometimes) $n \models R=S$ iff $\exists m. (n, m) \in R \cap S$. 
Operators on node sort are very familiar

- atomic tests
- test for being in the domain of a relation. (just like $\exists R. F$)
- closed under the booleans.
- (sometimes) \( n \models R = S \) iff \( \exists m. (n, m) \in R \cap S \).
  - term-definable from \( w \models R^{\text{loop}} \) iff \( (w, w) \in R \).
  - \( R = S \equiv (R; S^{-1})^{\text{loop}} \)
Primitive relations are tree relations

• **down, up, left, right**

• their transitive closures: descendant, ancestor, ...

• often syntactic sugar: following = ancestor*/right+/descendant*

• stay relation with a test:
Operators on path sort are also very familiar

- **Regular operators**: union, concatenation, Kleene closure
- **Boolean operators**: intersect and except
- **Variables and binders**: as in hybrid logic.
  - for $x$ in PATH1 return PATH2
  - **Meaning**: \( \downarrow y.\text{PATH1} / \downarrow x. @y/\text{PATH2} \)
Immediate relations to known formalisms

- node and path-formulas of PDL
- almost all operators can be found in some DL-language
- **Trees**: CTL, tree logics of [Blackburn, de Rijke, Meijer-Viol ’96]
- without Kleene *, all languages are inside **FO**.
Real life complications (1)

- Two syntaxes

- Unix path style:

  `/book//section[./paragraph[contains(.,''XML'')]]`

- Official style:

  `/child::book/descendant::section[child::paragraph[contains(.,''XML'')]]`

- Unix style only “up and down”. Official style: everything.
Real life complications (2)

XPath has many uses and interpretations.

1. Path formula denotes binary relation
   when used for navigation within other languages

2. path formula denotes set of nodes
   • when used as a stand-alone query language
   • Meaning of PATH is range of PATH.
   • Natural with /PATH (all nodes reachable by PATH from the root)

3. Path formula denotes a set of trees
   • XPath used as a constraint language
   • “all trees having a PATH from the root”
Example

• Task Express the tree-like interview model in XPath.

• For $N$ a node-formula (“modal formula”), $N$ holds everywhere iff the root starts path

\[ \text{not //*[not N]} \text{.} \]

$Q \rightarrow A^+$
Example

- **Task** Express the tree-like interview model in XPath.

- For $N$ a node-formula ("modal formula"), $N$ holds everywhere iff the root starts path

  $$\text{.}[\not /\![\not N]].$$

$Q \rightarrow A^+$

$Q$ and (not child::A or child::*[not A])
Example

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last Q without A
Example

- **Task** Express the tree-like interview model in XPath.

- For \( N \) a node-formula ("modal formula"), \( N \) holds everywhere iff the root starts path

\[
.\text{[not //*[not N]].}
\]

\( Q \rightarrow A^+ \)  
Q and (not child::A or child::*[not A])

\( \text{last Q without A} \)  
Q and not right::Q and child::A
Real life benefits

- Firefox and IE support XPath.
- Fast free XPath evaluators (Saxon, Libxslt)
- Good editors for XPath available
  - syntax highlighting
  - help with debugging
  - evaluation on XML docs
XPath practice

We define two information needs in terms of XPath.

1. a descendant with lots of specific ancestors along the way

2. question-answer pairs
Practice 1

Q return all q descendants of current node
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A descendant::q or .//q
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Q return all q descendants reachable through $p_1, \ldots, p_n$ nodes
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Q return all q descendants of current node

A descendant::q or .//q

Q return all q descendants reachable trough $p_1, \ldots, p_n$ nodes

A1 .//$p_1$//q intersect ... intersect .//$p_n$//q
Practice 1

Q return all q descendants of current node

A descendant::q or .//q

Q return all q descendants reachable through p\(_1\), \ldots, p\(_n\) nodes

A1 .//p\(_1\)//q intersect \ldots intersect .//p\(_n\)//q

A2 big union for all permutations \(\rho\) of 1, \ldots, n of

\[
.//p_{\rho(1)}///p_{\rho(2)}/\ldots//p_{\rho(n)}///q
\]
Practice 2: question-answers pairs

- Flat (QA+) models

- Find an XPath expression $x/\ldots$ which returns
  - when $x$ is bound to a Q node
  - all following A nodes until the next Q.

```
$x$

... QAA Q AAAQAQAAAAA ...
```
Kleene style

\$x/(\text{right}:A)^+.\$

- $(.)^+$ is the transitive closure operator.
- But $(.)^+$ is not available (and not expressible) in W3C XPath dialects (because that is just FO).
Tarski style

$x/((\text{following} - \text{sibling} :: A \text{ except} \\
\text{following} - \text{sibling} :: Q/\text{following} - \text{sibling} :: A))$

- Expressible in XPath with Booleans on path expressions [Hidders, 2003]
Frege (or first-order) style

$x$/following-sibling::A[  not
  preceding-sibling::Q/preceding-sibling::Q[. is $x]]$

- Uses variables bound to nodes
- Test . is $x$ is the hybrid logic variable test.
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Expressivity questions on trees

Rabin’s theorem sets a clear upper bound:

$$\text{MSO} = \text{tree automata} = \text{regular tree languages} = \text{decidable}.$$ 

Questions we will survey:

- expressivity relative to yardsticks
- succinctness
- semantic characterizations

Signature of the languages:

- equality, unary predicates for nodes, child, descendant, right, right+
Four XPath dialects

Four flavours of XPath strictly below MSO [ten Cate, M. 2007 survey]

Core XPath $\approx$ PDL without *

XPath 2.0 no vars $\approx$ Boolean modal logic $\approx$ Core XPath plus booleans on paths

XPath 2.0 $\approx$ hybrid Boolean modal logic

Regular XPath PDL with the four one-step tree relations.
Characterization of Core XPath

- On unary trees (= the line), this is Prior’s temporal logic with $F$ and $P$.

- Kamp’s theorem ’68 not enough to capture $\text{FO}(x)$ on the line.

- [Etessami, Vardi and Wilke ’97]: expressive power is exactly $\text{FO}_2(x)$, with an exponential succinctness gap.
  - “any two nodes that agree on $p_1, \ldots, p_n$ also agree on $p_0$”
  - linear constraint in $\text{FO}_2$, exponential in $\text{TL}$.

- Generalizes to sibling-ordered trees and Core XPath.
Core XPath plus booleans on paths

- Kamp’s thm on unary trees: \( \text{FO}(x) = \text{FO}_3(x) \).

- [M. 2005]: Generalizes to XML-trees and paths: \( \text{FO}(x,y) = \text{FO}_3(x,y) \)

- Tarski’s thm: \( \text{FO}_3(x, y) = \text{Tarski relation algebras} \).

- on trees: Tarski relation algebras = Core XPath plus booleans on paths

- Core XPath plus booleans on paths = \( \text{FO}(x,y) \) on XML trees.
Regular XPath

- Captures $FO(x, y)$ (because it captures “since and until”).

- [ten Cate 06] With additional loop it captures $FO^*(x, y)$.

\[ T, x \models R^{\text{loop}} \iff T, (x, x) \models R. \]

- [ten Cate, Segoufin 08] With additional subtree relativization it captures FO extended with monadic TC.

\[ T, x \models W\phi \iff T_x, x \models \phi. \]

- [ten Cate, Segoufin 08] Both are strictly less expressive than MSO.
## Summary

<table>
<thead>
<tr>
<th>XPath dialect</th>
<th>Core XPath 1.0 $\nsubseteq$ Variable-free Core XPath 2.0 $\equiv$ Core XPath 2.0 $\nsubseteq$ Regular XPath $\approx$ FO-dialect $\equiv$ Core XPath 2.0 $\nsubseteq$ Regular XPath $\approx$ FO-tree $\equiv$ Core XPath 2.0 $\nsubseteq$ Regular XPath $\approx$ FO-tree $^*$</th>
</tr>
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<tr>
<td>Equivalent FO-dialect</td>
<td>$\exists FO_{\text{tree}}^{\text{mon}}$ (exponential succinctness gap) $\equiv FO_{\text{tree}}$ (at least exponential succinctness gap) (no succinctness gap: linear translations) $\equiv FO_{\text{tree}}^*$ (non-elementary succinctness gap)</td>
</tr>
</tbody>
</table>

*Partial Datalog on trees.*
Semantic characterizations

• class of trees $C$ is definable in $L$ iff $C$ is closed under . . .

• Useful for inexpressivity results.

• Real-life languages (W3C standards) often have practical constraints with unexpected theoretical effects

• DTD’s: must be deterministic

  $(a+b)^*a(a+b)$ is not expressible by a DTD [Brüggemann-Klein Wood 98]

• XML schema’s must be single-typed specialized DTD’s [Murata, Lee, Mani ’01]
Characterization of single type SDTD

- [Martens, Neven, Schwentick 05] For $T$ a regular tree language, $T$ is definable by a single type SDTD iff $T$ is closed under ancestor-guarded subtree exchange.

**Ancestor-Guarded Subtree Exchange**

$T$ a regular tree language
(QA+) + Q is not definable on hierarchical models

- Interviews ending in a Q without an A.
- We could not find a DTD specifying this in the hierarchical model.
- Now we can prove it:
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Complexity questions: evaluation

- **Model checking.** Validation, querying

  **Input** Tree, node(s), formula. **Output** Boolean

- **PSPACE complete for FO. PTIME for fixed variable FO.**

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<tr>
<th>Fragment</th>
<th>Evaluation complexity</th>
<th>References</th>
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<td>Core XPath</td>
<td>PTime (linear)</td>
<td>[Gottlob, Koch, Pichler 02]</td>
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<tr>
<td>Core XPath 2 no vars</td>
<td>PTime (quadratic)</td>
<td>(from FO)</td>
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<tr>
<td>Core XPath 2</td>
<td>Pspace</td>
<td>(from FO)</td>
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<tr>
<td>Regular XPath</td>
<td>PTime (linear)</td>
<td>(from PDL)</td>
</tr>
<tr>
<td>Regular XPath+</td>
<td>PTime (linear)</td>
<td>[Gottlob Koch 04]</td>
</tr>
<tr>
<td>TMNF tests (=MSO)</td>
<td></td>
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</tbody>
</table>
Complexity: Static analysis

- Satisfiability, equivalence, ...

- **Decidable for MSO.** Non-elementary hard already for FO on unary trees [Rabin; Meyer]

- Complexity overview [ten Cate, Lutz, 2007]
  - Satisfiability.
  - Lower bound is EXPTIME, already for Core XPath
  - Small language extensions may yield large leaps in complexity
XPath: (P)DL on trees.
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XML language research and (P)DL: close relations

- both rooted in KR
- trees as fundamental models
- strong emphasis on working systems
- huge tables with acronyms and complexity classes ;-}
strong DL–XML interplay

- KR aspects

- Data integration and mediation [Halevy, Rome school] (certain answers are hard to compute)

- Design, maintenance, reuse, integration of ontologies is daily headache for XML/web-engineers

- DL’s research on modularity of TBoxes [Manchester school] seems useful.
Thank you
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Thank you